MATHEMATICS MODEL QUESTION PAPERS FOR 2020-21, 2021-22 & 2022-23 ADMITTED BATCHES

ADIKAVI NANNAYA UNIVERSITY :: RAJAMAHENDRAVARAM
B.A/B.Sc Mathematics Syllabus (w.e.f : 2020-21 A.Y)

MODEL QUESTION PAPER (Sem-End)
B.A./B.Sc. DEGREE EXAMINATIONS

Semester - I

Course-1: DIFFERENTIAL EQUATIONS

Time: 3Hrs **Max.Marks:75M**

SECTION - A

Answer any FIVE questions. **5 X 5 M=25 M**

1. Solve $(1+e^{x/y})dx + e^{x/y}\left(1-\frac{x}{y}\right)dy = 0$
2. Solve $(y - e^{sin^{-1}x})\frac{dx}{dy} + \sqrt{1-x^2} = 0$
3. Solve sin px cos y = cos px siny + p.
4. Solve $[D^2 - (a+b)D + ab]y = 0$
5. Solve $(D^2 - 3D + 2) = \cosh x$
6. Solve $(D^2 - 4D + 3)y = \sin 3x \cos 2x$.
7. Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = 8e^{3x}\sin 2x$.
8. Solve $x^2y'' - 2x(1+x)y' + 2(1+x)y = x^3$

SECTION - B

Answer ALL the questions. **5 X 10 M = 50 M**

9. (a) Solve $x\frac{dy}{dx} + y = y^2 \log x$.

(Or)

(b) Solve $\left(y + \frac{y^3}{3} + \frac{x^2}{2}\right)dx + \frac{1}{4}(x + xy^2)dy = 0$

10. (a) Solve $p^2 + 2py\cot x = y^2$.

(Or)

(b) Solve $y + Px = P^2x^4$

11. (a) Solve$(D^3 + D^2 - D - 1)y = \cos 2x$.11

(OR)

(b) Solve $(D^2 - 3D + 2)y = \sin e^{-x}$.

12. (a) Solve $(D^2 - 2D + 4)y = 8(x^2 + e^{2x} + \sin 2x)$

(Or)

(b) Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = xe^x \sin x$

13. (a) Solve $(D^2 - 2D)$ y = e^x sin x by the method of variation of parameters.

(Or)

B.A/E (b) Solve $3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x$

MODEL QUESTION PAPER (Sem-End)
B.A./B.Sc. DEGREE EXAMINATIONS

Semester - II

Course-2: THREE DIMENSIONAL ANALYTICAL SOLID GEOMETRY

Time: 3Hrs **Max.Marks:75M**

SECTION - A

Answer any FIVE questions. **5 X 5 M=25 M**

1. Find the equation of the plane through the point (-1,3,2) and perpendicular to the planesx+2y+2z=5 and 3x+3y+2z=8.
2. Find the bisecting plane of the acute angle between the planes 3x-2y-6z+2=0,-2x+y-2z-2=0.
3. Find the image of the point (2,-1,3) in the plane 3x-2y+z =9.

4. Show that the lines $2x + y - 4 = 0 = y + 2z$ and $+ \ 3z - 4 = 0$, $2x + 5z - 8 = 0$ are coplanar.

5. A variable plane passes through a fixed point (a, b, c). It meets the axes in A, B, C. Show that thecentre of the sphere OABC lies on $ax^{-1}+by^{-1}+cz^{-1}=2$.
6. Show that the plane 2x-2y+z+12=0 touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$ and find the pointof contact.
7. Find the equation to the cone which passes through the three coordinate axes and the lines $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ *and* $\frac{x}{2} = \frac{y}{1} = \frac{z}{1}$
8. Find the equation of the enveloping cone of the sphere $x^2 + y^2 + z^2 + 2x - 2y = 2$ withits vertex at (1, 1, 1).

SECTION - B

Answer ALL the questions. **5 X 10 M = 50 M**

9. (a) A plane meets the coordinate axes in A, B, C. If the centroid of ABc (a,b,c), show that theEquation of the plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$.

(OR)

(b) A variable plane is at a constant distance p from the origin and meets the axes in A,B,C. Show thatThe locus of the centroid of the tetrahedron OABC is $x^{-2}+y^{-2}+z^{-2}=16p^{-2}$.

10. (a) Find the shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1};\ \frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$.

(OR)

(b) Prove that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4};\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ are coplanar. Also find their point ofintersection and the plane containing the lines.

11. (a) Show that the two circles $x^2+y^2+z^2-y+2z=0$, $x-y+z=2$; $x^2+y^2+z^2+x-3y+z-5=0$, $2x-y+4z-1=0$ lie on the same sphere and find its equation.

(OR)

(b) Find the equation of the sphere which touches the plane $3x+2y-z+2=0$ at $(1,-2,1)$ and cuts orthogonallyThe sphere $x^2+y^2+z^2-4x+6y+4=0$.

12. (a) Find the limiting points of the coaxial system of spheres $x^2+y^2+z^2-8x+2y-2z+32=0$, $x^2+y^2+z^2-7x+z+23=0$.

(OR)

(b) Find the equation to the cone with vertex is the origin and whose base curve is $x^2+y^2+z^2+2ux+d=0$.

13 (a) Prove that the equation $\sqrt{fx}\pm\sqrt{gy}\pm\sqrt{hz}=0$ represents a cone that touches the coordinatePlanes and find its reciprocal cone.

(OR)

(b) Find the equation of the sphere $x^2+y^2+z^2-2x+4y-1=0$ having its generators parallel to the line $x=y=z$.

MODEL QUESTION PAPER (Sem-End)
B.A./B.Sc. DEGREE EXAMINATIONS
Semester - III
Course-3: ABSTRACT ALGEBRA

Time: 3Hrs **Max.Marks:75M**

SECTION – A

Answer any FIVE questions. **5 X 5 M=25 M**

1. Show that the set G=$\{x/x = 2^a 3^b \text{ and } a, b \in Z\}$ is a group under multiplication
2. Define order of an element. In a group G, prove that if $a \in G$ then $O(a) = O(a)^{-1}$.
3. If H and K are two subgroups of a group G, then prove that HK is a subgroup ⇔ HK=KH
4. If G is a group and H is a subgroup of index 2 in G then prove that H is a normal subgroup.
5. Examine whether the following permutations are even or odd

   i) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 6 & 1 & 4 & 3 & 2 & 5 & 7 & 8 & 9 \end{pmatrix}$ ii) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 4 & 5 & 6 & 7 & 1 \end{pmatrix}$
6. If f is a homomorphism of a group G into a group G', then prove that the kernel of f is a normal of G.
7. Prove that the characteristic of an integral domain is either prime or zero.
8. Define a Boolean Ring and Prove that the Characteristic of a Boolean Ring is 2.

SECTION - B

Answer ALL the questions. **5 X 10 M = 50 M**

9. a) Show that the set of n^{th} roots of unity forms an abelian group under multiplication.

   (Or)

   b) In a group G, for $a, b \in G$, O(a)=5, b ≠ e and $aba^{-1} = b^2$. Find O(b).

10. a) The Union of two subgroups is also a subgroup □ one is contained in the other.

    (Or)

    b) State and prove Langrage's theorem.

11. a) Prove that a subgroup H of a group G is a normal subgroup of G iff the product of two right cosets of H in G is again a right coset of H in G.

    (Or)

    b) Define Normal Subgroup. Prove that a subgroup H of a group G is normal iff $xHx^{-1} = H \; \forall \, x \in G$.

12. a) State and prove fundamental theorem of homomorphisms of groups.

    (Or)

    b) Let S_n be the symmetric group on n symbols and let A_n be the group of even permutations. Then show that A_n is normal in S_n and O(A_n) $= \frac{1}{2}(n!)$

13. a) Prove that every finite integral domain is a field.

    (Or)

    b) Let S be a non empty sub set of a ring R. Then prove that S is a sub ring of R if and only if a-b ∈ S and ab ∈ S for all a, b ∈ S.

(b) If f: [a, b] → R is continuous on [a, b] then prove that f is bounded on [a, b]

12. (a) Using Lagrange's theorem, show that $x > \log(1+x) > \frac{x}{(1+x)} \; \forall \, x > 0$.

(OR)

(b) State and prove Cauchy's mean value theorem...

13. (a) State and prove Riemman's necessary and sufficient condition for R- integrability.

(OR)

(b) Prove that $\frac{\pi^3}{24} \leq \int_0^{\pi} \frac{x^2}{5+3\cos x} dx \leq \frac{\pi^3}{6}$

MODEL QUESTION PAPER (Sem-End)

B.A./B.Sc. DEGREE EXAMINATIONS

Course-4: REAL ANALYSIS

Time: 3Hrs **Max.Marks:75M**

SECTION - A

Answer any FIVE questions. **5 X 5 M=25 M**

1. Prove that every convergent sequence is bounded.
2. Examine the convergence of $\frac{1}{1.2} - \frac{1}{3.4} + \frac{1}{5.6} - \frac{1}{7.8} + \cdots$
3. Test the convergence of the series $\sum_{n=1}^{\infty}(\sqrt[3]{n^3+1} - n)$.
4. Examine for continuity of the function f defined by $f(x) = |x| + |x-1|$ at x=0 and 1.
5. Show that $f(x) = x\sin\frac{1}{x},\ x \neq 0; f(x) = 0, x = 0$ is continuous but not derivable at x=0.
6. Verify Rolle's theorem for the function $f(x) = x^3 - 6x^2 + 11x - 6$ on $[1,3]$.
7. If $f(x) = x^2\ \forall\ x \in [0,1]$ and $p = \{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\}$ then find L(p, f) and U(p, f).
8. Prove that if $f: [a,b] \to R$ is continuous on [a, b] then f is R- integrable on [a, b].

SECTION –B

Answer ALL the questions. **5 X 10 M = 50 M**

9. (a) If $S_n = 1 + \frac{1}{2!} + \frac{1}{3!} + - - - - - - - + \frac{1}{n!}$ then show that $\{S_n\}$ converges.

(OR)

(b) State and prove Cauchy's general principle of convergence.

10. (a) State and Prove Cauchy's nth root test.

(OR)

(b) Test the convergence of $\sum \frac{x^n}{x^n + a^n}$ $(x > 0, a > 0)$

11. (a) Let f: R→ R be such that

$$f(x) = \frac{\sin(a+1)x + \sin x}{x} \text{ for } x < 0$$

$$= c \quad \text{for } x = 0$$

$$= \frac{(x+bx^2)^{1/2} - x^{1/2}}{bx^{3/2}} \text{ for } x > 0$$

Determine the values of a, b, c for which the function f is continuous at x=0.

(OR)

(b) If f: [a, b] → R is continuous on [a, b] then prove that f is bounded on [a, b]

12. (a) Using Lagrange's theorem, show that $x > \log(1+x) > \frac{x}{(1+x)} \forall x > 0$.

(OR)

(b) State and prove Cauchy's mean value theorem...

13. (a) State and prove Riemman's necessary and sufficient condition for R- integrability.

(OR)

(b) Prove that $\frac{\pi^3}{24} \leq \int_0^{\pi} \frac{x^2}{5+3\cos x} dx \leq \frac{\pi^3}{6}$

MODEL QUESTION PAPER (Sem-End)

B.A./B.Sc. DEGREE EXAMINATIONS

Semester -IV

Course-5: LINEAR ALGEBRA

Time: 3Hrs **Max.Marks:75M**

SECTION - A

Answer any FIVE questions. **5 X 5M=25 M**

1. Let p, q, r be fixed elements of a field F. Show that the set W of all triads (x, y, z) of elements of F, such that px+qy+rz=0 is a vector subspace of $V_3(R)$.
2. Define linearly independent &linearly dependent vectors in a vector space. If α, β, γ are linearly independent vectors of V(R) then show that $\alpha+\beta, \beta+\gamma, \gamma+\alpha$ are also linearly independent.
3. Prove that every set of $(n+1)$ or more vectors in an n dimensional vector space is linearly dependent.
4. The mapping T : V3(R) V3(R) is defined by T(x,y,z) = (x-y,x-z). Show that T is a linear ransformation.
5. Let $T: R^3 \to R^2$ and $H: R^3 \to R^2$ be defined by T (x, y, z)= (3x, y+z) and H (x, y, z)= (2x-z, y). Compute i) T+H ii) 4T-5H iii) TH iv) HT.
6. If the matrix A is non-singular, show that the eigen values of A^{-1} are the reciprocals of the eigen values of A.
7. State and prove parallelogram law in an inner product space V(F).
8. Prove that the set S = $\left\{\left(\frac{1}{3},\frac{-2}{3},\frac{-2}{3}\right),\left(\frac{2}{3},\frac{-1}{3},\frac{2}{3}\right),\left(\frac{2}{3},\frac{2}{3},\frac{-1}{3}\right)\right\}$ is an orthonormal set in the inner product space $R^3(R)$ with the standard inner product.

SECTION - B

Answer ALL the questions. **5 X 10 M = 50 M**

9. (a) Define vector space. Let V (F) be a vector space. Let W be a non empty sub set of V. Prove that the Necessary and sufficient condition for W to be a subspace of V is $a, b \in F$ and $\alpha, \beta \epsilon V => a\alpha + b\beta \epsilon W$

(OR)

(b) Prove that the four vectors (1,0,0), (0,1,0), (0,0,1) and (1,1,1) of $V_3(C)$ form linearly dependent set, but any three of them are linearly independent.

10. (a) Define dimension of a finite dimensional vector space. If W is a subspace of a finite Dimensional vector space V (F) then prove that W is finite dimensional and dim W $\leq n$.

(OR)

(b) If W be a subspace of a finite dimensional vector space V(F) then Prove that

. $\dim {}^{V}/_{W} = \dim V - \dim W$

11. (a) Find T (x, y, z) where $T: R^3 \rightarrow R$ is defined by T (1, 1, 1) =3, T (0, 1, -2) =1,T (0, 01) = -2

(OR)

(b) State and prove Rank Nullity theorem.

12. (a) Find the eigen values and the corresponding eigen vectors of the matrix A= $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$

(OR)

(b) State and prove Cayley-Hamilton theorem.

13. (a) State and prove Schwarz's inequality in an Inner product space V(F).

(OR)

(b) Given $\{(2,1,3), (1,2,3), (1,1,1)\}$ is a basis of R^3(R).Construct an orthonormal basis using Gram-Schmidorthogonalisation process.

ADIKAVI NANNAYYA UNIVERSITY :: RAJAMAHENDRAVARAM
CBCS/ SEMESTER SYSTEM
(W. e. f 2020 – 21 Admitted Batch)
B. A./B. Sc. MATHEMATICS
COURSE – VI(A), NUMERICAL METHODS.

MATHEMATICS MODEL PAPER

Max. Marks: 75M **Time: 3Hrs**

SECTION – A

Answer any FIVE questions. Each question carries FIVE marks. **5 X 5 M = 25 M**

1) Find the function whose first difference is $9x^2 + 11x + 5$.

2) Find the missing term in the following table

x	0	1	2	3	4
y	1	1.5	2.2	3.1	4.6

3) If $f(x) = \frac{1}{x^2}$ then find the divided differences $f(a, b)$ and $f(a, b, c)$.

4) Using Gauss forward interpolation formula to find f(2.5) from the following table.

x	1	2	3	4
f(x)	1	8	27	64

5) Derive the derivative $\left(\frac{dy}{dx}\right)_{x = x_0}$ by using Newton's backward interpolation formula.

6) Find $\frac{dy}{dx}$ at $x = 0$, using the table

x	0	2	4	6	8	10
f(x)	0	12	248	1284	4080	9980

7) Evaluate the integral $\int_0^6 \frac{dx}{1+x}$ by using Simpson's $\frac{1}{3}$ rule.

8) Using Taylor's series method, solve the equation $\frac{dy}{dx} = x^2 + y^2$ for $x = 0.4$, given that $y = 0$ when $x = 0$.

SECTION – B

Answer any ALL questions. Each question carries TEN marks. **5 X 10 M = 50 M**

9 a) State and Prove Newton's forward interpolation formula.

OR

9 b) Show that i) $\mu^2 = 1 + \frac{1}{4}\delta^2$ and ii) $1 + \mu^2\delta^2 = \left(1 + \frac{1}{2}\delta^2\right)^2$

10 a) State and prove Bessel's formula.

OR

10 b) Using Lagrange's formula fit a polynomial to the following data and hence find f(1).

x	-1	0	2	3
f(x)	8	3	1	12

11 a) Derive the derivatives $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = x_0$ by using Stirling's interpolation formula.

OR

11 b) Compute $f^1(4)$ and $f^1(5)$ from the following table

x	1	2	4	8	10
f(x)	0	1	5	21	27

12 a) State and prove General Quadrature Formula.

OR

12 b) Evaluate the integral $\int_0^6 \frac{dx}{1+x^3} dx$ by using Weddle's rule.

13 a) Use Runge – Kutta method to evaluate y (0.1) and y (0.2) given that $y^1 = x + y$, initial condition $y(0) = 1$.

OR

13 b) Given $\frac{dy}{dx} = x + y$ with initial condition $y(0) = 1$. Find y(0.05) and y(0.1), correct to 6 decimal places by using Euler's modified method.

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B.A./B.Sc., FIFTH SEMESTER MATHEMATICS MODEL PAPER

7A: MATHEMATICAL SPECIAL FUNCTIONS

(w. e. f. 2020-21 admitted batch)

TIME: 3hrs MAX.MARKS:75

SECTION- A

Answer any **FIVE** questions. Each question carries 5 marks. 5 X 5 = 25 Marks

1. Evaluate $\int_0^2 \frac{x^2 dx}{\sqrt{(2-x)}}$
2. Show that $\Gamma\left(\frac{1}{2}+x\right)\,\Gamma\left(\frac{1}{2}-x\right) = \frac{\pi}{Cos\ \pi x}$
3. If the power series $\sum a_n x^n$ is such that $a_n \neq 0$ for all n and $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{R}$ then prove that $\sum a_n x^n$ is convergent for $|x| < R$ and divergent for $|x| > R$
4. Prove that $H_n''(x) = 4n(n-1)\,H_{n-1}(x)$
5. Prove that $H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}$
6. Prove that $P_n(-x) = (-1)^n P_n(x)$
7. Prove that $P_n'(1) = \frac{1}{2}n(n+1)$
8. Prove that $J_{-n}(x) = (-1)^n J_n(x)$ where n is a positive integer

SECTION -B

Answer any **FIVE** questions. Each question carries 10 marks. 5 X 10 = 50 Marks

9(a). Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

OR

9(b). Prove that $2^n\Gamma\left(n+\frac{1}{2}\right) = 1.3.5\ \ldots.(2n-1)\sqrt{\pi}$ *where n is a* positive integer

10(a). Solve $y' - y = 0$ by power series method

OR

10(b). Find the power series solution in powers of (x-1) of the initial value problem

$$xy'' + y' + 2y = 0, y(1) = 1, y'(1) = 2.$$

11(a). Prove that $H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}\left(e^{-x^2}\right)$

OR

11(b). Prove that $2xH_n(x) = 2n\, H_{n-1}(x) + \, H_{n+1}(x)$

12(a). Prove that $(1 - 2xh + h^2)^{-1/2} = \sum_{n=0}^{\infty} h^n P_n(x)$

OR

12(b). $\int_{-1}^{1} P_m(x)\, P_n(x) dx = 0 \; if \; m \neq n$

13(a). $xJ_n'(x) = n\, J_n(x) - x\, J_{n+1}(x)$

OR

13(b). Show that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}}\, Cos\, x$

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